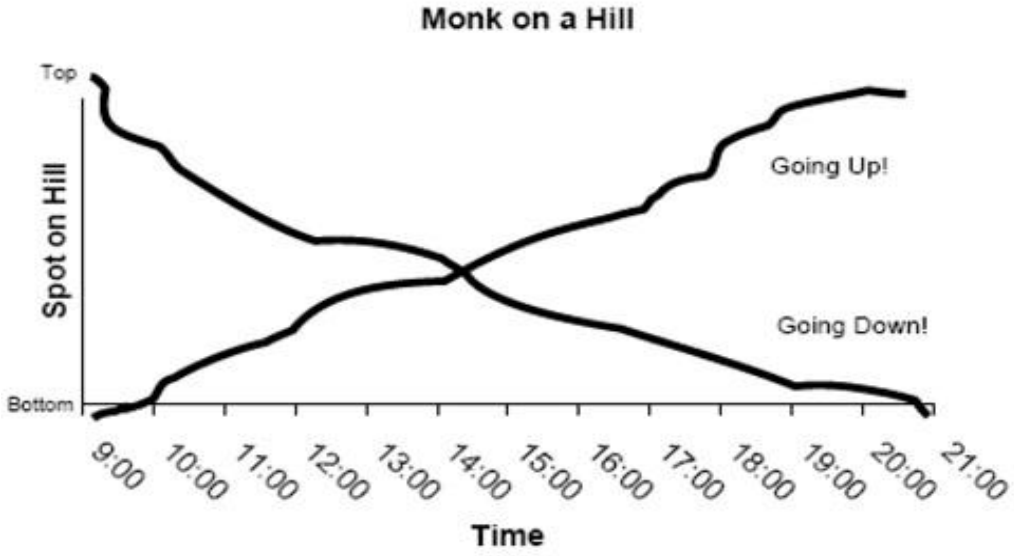
Brainteaser: A Monk on a Hill

(Excerpted from Prof. Edward H. Kaplan's Lecture Notes for Individual Problem Framing, Fall 2006)

At 9:00 in the morning, a monk starts to climb a hill, and spends the next twelve hours hiking to the top. He reaches the top at 9:00 in the evening, and spends the night there in meditation. The next morning, at 9:00 he starts to descend along the same path he used to climb the hill the day before. Taking his time, he reaches the bottom at 9:00 in the evening of the second day. Your task: show that there is some spot on the hill such that the monk is at that spot at the *same* time on *each* day.

Hmmm — we need to make sure we understand the situation. The monk starts climbing the hill at 9:00 AM and reaches the top at 9:00 PM. That means that at any particular time in between, the monk is *somewhere* on the hill. Similarly, on the second day the monk starts walking down at 9:00 AM and reaches the bottomat 9:00 PM — and at any particular time in between, the monk is *somewhere* on the hill.

So do we understand what is going on? Can you draw a picture of the situation? For example, can you draw a picture that shows where the monk is at any time on the first day? On the second day? Can you draw one picture that shows the monk going up the hill on the first day and descending on the second day? How about the picture below?



Gee — all the picture above does is repeat the problem — it says that the monk starts at the bottom and ends at the top when he goes up, and starts at the top and ends at the bottom when he goes down. Of course, the picture also shows you where the monk is at any time on both days, and how about that — the up and the down curves cross — which means that there is a spot on the hill the monk visits at the same time on each day!

What is cute (and special) about this problem is that once you understand the situation clearly, the answer simply leaps out at you. No extra work!

Now drawing a picture was not the only way to understand this situation. You might have simply said "Hmmm — the monk goes up on day one, and down on day two, and both times takes the same path. What if I change the problem to one of two monks, with the second monk starting at the top on the first day and walking exactly as the first monk would on the second day in the real problem? *Gee — at some point the two monks are going to pass each other! Hi Monk! Hi Monk!"* Problem solved.

Or you might have thought of an analogy. "Hmmm — this is just like a cable car going up and coming down, except on different days. But we all know that cable cars pass each other along the way, so there's a point where both cable cars are at the same time!" Or a different analogy — "hmmmm, this is sort of like lighting a candle at both ends — it would take the same time for an entire candle to burn if I lit it at one end only, or at the other end only, but if I light it at both ends, at some point the flames will meet — hey, starting the candle at one end or the other is like starting the monk at the bottom or the top, and where the flames meet is the desired spot on the hill."

Take home message: when confronting a new problem, make sure that you understand it clearly. Sometimes that is enough to answer the question. Secondary lesson — drawing a picture or making an analogy or transforming the problem into something equivalent that you understand better is a good technique for exploring a new problem.