Technical Appendix: When To Fire Customers? Customer Cost Based Pricing

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May 2011

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Numerical Example

In the main text, we set $\delta = 0.9$, $v = 1.2$, s^L=0. For the low and high service cost heterogeneity cases, we set s^H=0.3 and s^H=1, respectively.

Several diagnostic metrics for the first and second periods are provided in Table A below. Customers who buy in the first period are referred to as "Old" customers and those who buy only in the second period are referred to as "New" customers. First, consistent with propositions 1 and 2 in the main paper, CCP leads to lower aggregate profits ($\Pi^{CCP} = 0.946$) than the benchmark profits without CCP ($\Pi^{NoCCP} = 1.047$) when the service cost heterogeneity is low ($\Delta s = 0.3$) ; but CCP has higher profit ($\Pi^{CCP} = 0.470 > \Pi^{NoCCP} = 0.466$) when the service cost heterogeneity is high ($\Delta s = 1$). One can also see the intuition about the ratchet effect being stronger when service cost hetergoenteity is low. When service cost heterogeneity is low ($\Delta s = 0.3$), second period prices rise by 57% for old customers ($p_1 = 0.49 \rightarrow p_2 = 0.77$). No distinction is made between the high and low cost types; only the purchase information is used in setting prices (as in Villas-Boas 2004). In contrast, when service cost heterogeneity is high ($\Delta s = 1$), both cost and purchase information are used. Second period prices rise by 65% for the high cost old customers ($p_1^H = 0.67 \rightarrow p_2^H = 1.1$), but only by 21% for the low cost customers ($p_1^L = 0.67 \rightarrow p_2^L = 0.81$). The weaker ratcheting effect for the old low cost customers coupled with the price discrimination effect between high and low cost customers makes CCP more profitable.

Second, the optimal retention strategy depends on the service cost heterogeneity. When the service cost heterogeneity is low, the average retention rate is 100% because there is no reason to "fire" customers. The average retention rate declines to 62.5% when the service cost heterogeneity is high, but this lower retention is differentiated by high and low cost customers. 100% of the low cost customers are retained, but only 25% of the high cost customers are retained. In contrast to general exhortations to raise retention rates across customers, our results demonstrate that optimal retention rates should be managed based on customer characteristics. Firms should lower retention rates among its higher cost customers in order to obtain a more favorable mix of low to high cost customers.

Table A-1: Low Service Cost Heterogeneity ($\Delta s = 0.3$)

	Period 1						
	Old		New		Benchmark 1		Benchmark 2
	Low Cost	High Cost	Low Cost	High Cost	Old	New	
Price	0.49	0.49			0.49		0.675
Margin	0.49	0.19			0.34		0.525
Quantity	0.43	0.43			0.86		1.05
Customer Share	50%	50%					

	Period 2						
	Old		New		Benchmark 1		Benchmark 2
	Low Cost	High Cost	Low Cost	High Cost	Old	New	
Price	0.77	0.77	0.46	0.46	0.77	0.46	0.675
Margin	0.77	0.47	0.46	0.16	0.62	0.31	0.525
Quantity	0.43	0.43	0.31	0.31	0.43	0.31	1.05
Customer Share	29%	29%	21%	21%			
Retention	100%	100%					

$$\Pi^{CCP} = 0.946 < \Pi^{NoCCP} = 1.047$$

*Benchmark 1: traditional behavior-based price discrmination based only on the past purchase history.

†Benchmark 2: the case without price discrimination in which the firms uses neither the past purchase information nor the customer cost type information.

Table A-2: High Service Cost Heterogeneity ($\Delta s = 1$)

	Period 1						
	Old		New		Benchmark 1		Benchmark 2
	Low Cost	High Cost	Low Cost	High Cost	Old	New	
Price	0.67	0.67			0.73		0.85
Margin	0.67	-0.33			0.23		0.35
Quantity	0.39	0.39			0.571		0.70
Customer Share	50%	50%					

	Period 2						
	Old		New		Benchamrk 1		Benchmark 2
	Low Cost	High Cost	Low Cost	High Cost	Old	New	
Price	0.81	1.1	0.65	0.65	0.91	0.71	0.85
Margin	0.81	0.1	0.65	-0.35	0.41	0.21	0.35
Quantity	0.39	0.1	0.15	0.15	0.571	0.42	0.70
Customer Share	49%	13%	19%	19%			
Retention	100%	25%					

$$\Pi^{CCP} = 0.470 > \Pi^{NoCCP} = 0.466$$

*Benchmark 1: traditional behavior-based price discrmination based only on the past purchase history.

†Benchmark 2: the case without price discrimination in which the firms uses neither the past purchase information nor the customer cost type information.

Analysis for endogenous case when τ is sufficiently large ($\tau \geq \tau^{IC} = \frac{(2\Delta s-(v-s^H)\delta)\delta}{4(2-\delta)}$)

Recall we only consider the case of $\Delta s > \frac{\delta v_H^{max}}{2}$, where cost information was used by the monopolist in the *exogenous case*. This condition ensures that $\widehat{w}_1^L > \frac{v+s^L}{2}$ and $\widehat{w}_1^H < \frac{v+\tau+s^H}{2} - \tau$ are satisfied in equilibrium. We confirm that $\frac{v+s^L}{2} < \widehat{w}_1^L$ and $\frac{v+\tau+s^H}{2} - \tau > \widehat{w}_1^H$ are indeed satisfied when $\Delta s > \frac{\delta v_H^{max}}{2}$. Therefore, if (IC) constraint is not binding, the firm charges

$$p_2^L = \widehat{w}_1^L \text{ and } p_2^H = argmax_p(p-s^H)(v+\tau-p) = \frac{v+\tau+s^H}{2}, \tag{0.1}$$

and $D_2^L(p) = \min\{v-p_2^L, v-\widehat{w}_1^L\} = v-\widehat{w}_1^L$, $D_2^H(p) = \min\{v+\tau-p_2^H, v-\widehat{w}_1^H\} = v+\tau-\frac{v+\tau+s^H}{2} = \frac{v+\tau-s^H}{2}$. Hence, the monopolist does not serve all of the previous customers.

Like the exogenous model in the main paper, the firm sets price for customers who have not purchased in the first period using the first-order condition from Equation (15) in the main paper as follows:

$$p_2^O = \text{argmax}_p(p-s^H)(\widehat{w}_1^H - p + \tau) + (p-s^L)(\widehat{w}_1^L - p) = \frac{\widehat{w}_1^H + \widehat{w}_1^L + s^H + s^L + \tau}{4}, \tag{0.2}$$

We summarize the equilibrium outcomes in the following lemma.

Lemma 1. *When service demanded by consumers (and therefore cost-to-serve) is endogenous and the utility from service τ is sufficiently large that (IC-H) constraint will be always satisfied, the marginal customers in the first period differ by customer type:*

$$\widehat{w}_1^L = \frac{2v+2\overline{s}-\delta s^H}{4-\delta} + \frac{(2-\delta)\tau}{2(4-\delta)}, \quad \widehat{w}_1^H = \frac{2v+2\overline{s}-\delta s^H}{4-\delta} - \frac{(6-\delta)\tau}{2(4-\delta)}.$$

The equilibrium outcomes are as follows:

$$p_1 = \frac{(4v-(2-\delta)\tau)(2-\delta)+8\overline{s}-\Delta s(2\delta-\delta^2)}{4(4-\delta)},$$
$$p_2^H = \frac{v+s^H+\tau}{2}, \; p_2^L = \widehat{w}_1^L = \frac{2v+2\overline{s}-\delta s^H}{4-\delta} + \frac{(2-\delta)\tau}{2(4-\delta)}, \; \textit{and } p_2^O = \frac{\widehat{w}_1^H + \widehat{w}_1^L + s^H + s^L + \tau}{4}.$$

Proof. The marginal consumer in the first period, $\widehat{w}_1^j = \widehat{w}_1^j(p_1)$, can be calculated by using the fact that $p_2^O < p_1$ and the marginal consumer does not get any surplus in the second period if she already purchased it in the first period: $\widehat{w}_1^H + \tau - p_1 = \delta\left(\widehat{w}_1^H + \tau - p_2^O\right) \Leftrightarrow \widehat{w}_1^H = \frac{p_1 - \tau - \delta\left(p_2^O - \tau\right)}{1-\delta}$, and $\widehat{w}_1^L - p_1 = \delta\left(\widehat{w}_1^L - p_2^O\right) \Leftrightarrow \widehat{w}_1^L = \frac{p_1 - \delta p_2^O}{1-\delta}$.

Therefore, we can obtain the first period cutoff line for purchasing a product by using the fact that $p_2^O = \frac{\widehat{w}_1^H + \widehat{w}_1^L + s^H + s^L + \tau}{4}$: $\widehat{w}_1^H = \frac{2p_1 - \delta\overline{s}}{2-\delta} - \tau$, $\widehat{w}_1^L = \frac{2p_1 - \delta\overline{s}}{2-\delta}$.

In the first period, the monopolist maximizes the following total expected profit with a common discount factor $\delta < 1$:

$$\begin{aligned}\Pi(p_1) =&(p_1 - s^H)\left(v - \widehat{w}_1^H\right) + (p_1 - s^L)\left(v - \widehat{w}_1^L\right) + \delta\{(p_2^H - s^H)(v + \tau - p_2^H) + (p_2^L - s^L)(v - p_2^L) \\ &+ (p_2^O - s^H)(\widehat{w}_1^H + \tau - p_2^O) + (p_2^O - s^L)(\widehat{w}_1^L - p_2^O)\},\end{aligned}$$

where $\widehat{w}_1^H = \frac{2p_1 - \delta\overline{s}}{2-\delta} - \tau, \widehat{w}_1^L = \frac{2p_1 - \delta\overline{s}}{2-\delta}$, $p_2^L = \widehat{w}_1^L$, $p_2^H = \dfrac{v + \tau + s^H}{2}$.

Taking the first order condition gives us the first period price p_1:

$$p_1 = \frac{(4v - (2-\delta)\tau)(2-\delta) + 8\overline{s} - \Delta s(2\delta - \delta^2)}{4(4-\delta)}.$$

And the first period marginal consumer is now,

$$\widehat{w}_1^L = \frac{2v + 2\overline{s} - \delta s^H}{4-\delta} + \frac{(2-\delta)\tau}{2(4-\delta)}, \quad \widehat{w}_1^H = \frac{2v + 2\overline{s} - \delta s^H}{4-\delta} - \frac{(6-\delta)\tau}{2(4-\delta)}.$$

Hence,

$$p_2^H = \frac{v + s^H + \tau}{2}, \quad p_2^L = \widehat{w}_1^L = \frac{2v + 2\overline{s} - \delta s^H}{4-\delta} + \frac{(2-\delta)\tau}{2(4-\delta)}.$$

Further, we can check that $\dfrac{v + s^L}{2} < \widehat{w}_1^L$ and $\dfrac{v + \tau + s^H}{2} - \tau > \widehat{w}_1^H$ are satisfied in equilibrium when $\dfrac{\delta\left(v - s^H\right)}{2} < \Delta s$:

$$\widehat{w}_1^L - \frac{v + s^L}{2} = \frac{\delta(v - s^L) + 2(1-\delta)(s_H - s_L) + (2-\delta)\tau}{2(4-\delta)} > 0,$$

$$\widehat{w}_1^H - \left(\frac{v + \tau + s^H}{2} - \tau\right) = \frac{\delta(v - s^H) - 2(s_H - s_L) - 2\tau}{2(4-\delta)} < 0.$$

□

Unlike the exogenous case, where customer type is fixed, now the marginal customer in the first period from the high and low type differ in their willingness to pay. This is because the high type customer gets an extra utility τ from consuming the firm's augmented services.

From Lemma 1, we identify the τ-condition for (IC-H) constraint to be satisfied:

$$p_2^L \geq p_2^H - \frac{\tau}{\delta} \Leftrightarrow \frac{2v + 2\overline{s} - \delta s^H}{4-\delta} + \frac{(2-\delta)\tau}{2(4-\delta)} \geq \frac{v + s^H + \tau}{2} - \frac{\tau}{\delta} \tag{0.3}$$
$$\Leftrightarrow \tau \geq \tau^{IC} = \frac{(2\Delta s - (v - s^H)\delta)\delta}{4(2-\delta)}.$$

That is, when τ is sufficiently large ($\tau \geq \tau^{IC}$), the high type consumer will reveal his type in the first period by choosing a high level of service.

Results. *Suppose that the service cost heterogeneity is sufficiently large that* $\Delta s > \frac{\delta v_H^{max}}{2}$*, and the utility from service is large enough that* $\tau \geq \tau^{IC} = \frac{(2\Delta s - (v-s^H)\delta)\delta}{4(2-\delta)}$.

1. *The monopolist uses CCP and charges different prices to the H and L-type customers:* $p_2^H = \frac{v+s^H+\tau}{2} > p_2^L = \widehat{w}_1^L$.

2. *Further, the total profit with CCP is greater than the total profit without price discrimination* ($\Pi^{CCP} > \Pi^{NoPD}$) *when* Δs *becomes large.*

Proof. First, we calculate the profit under no discrimination. Without price discrimination, the monopolist simply maximizes the following per-period profit function $\Pi_t^S = (p^S - s^H)(v - p^S + \tau) + (p^S - s^L)(v - p^S)$. The optimal price is $p^S = \frac{2v+s^H+s^L+\tau}{4}$, and the total profit is $\Pi^S = (1+\delta)\left[\Pi_t^S\right] = (1+\delta)\frac{\left[(2v-s^H-s^L)^2+2(2v-3s^H+s^L)\tau+\tau^2\right]}{8}$. Using the results of p_1,p_2^H, and p_2^L in Lemma 3 of the main paper, we get

$\Pi^{CCP} = \frac{2v^2(4+\Omega)+(s^H)^2(2+\delta)^2+(s^L)^2\Omega+2s^L\Omega(2v-\tau)-2s^H(2+\delta)(4v-s^L(2-\delta))+\Omega(4v\tau-6s^H+\tau^2)}{8(4-\delta)}$, where $\Omega = 4 + (2-\delta)\delta$. It immediately follows that

$\Delta\Pi = \Pi^{CCP} - \Pi^S = \frac{\delta\left(2v^2(2-\delta)+4v\tau-(s^H)^2(1+2\delta)-(s^L)^2-s^L(4v-2\tau)-\tau^2+s^H(6s^L-4v(1-\tau)+6\tau\right)}{8(4-\delta)}$.

Therefore, $\Delta\Pi \geq 0$ if and only if $\Delta s \geq \frac{(v - s^L + \tau)\left(\sqrt{2(4-\delta)} - 2(1-\delta)\right) - (1+2\delta)\tau}{1+2\delta}$. Hence, as Δs becomes larger, $\Pi^{CCP} > \Pi^S$. □

When $\tau \geq \tau^{IC}$, the (IC) constraint for H-type is not binding. Therefore, the customers will reveal their types in the first period even under optimal prices that the monopolist would have charged when the customer type is fixed. Hence, the equilibrium outcome is consistent with the our main model where the customer types are exogeneous and CCP can increase a firm's profit.

Comparison between Price difference and Cost difference

Exogenous case:

1. When the heterogeneity is small: there is no price discrimination so that price difference (zero) is smaller than cost difference.

2. When the heterogeneity is large, price difference is always smaller than cost difference as follows:

$$p_H = \frac{v+s_H}{2},\ p = \frac{2v+2\overline{s}-\delta s_H}{4-\delta},$$
$$p_H - p_L = \frac{s_H(2+\delta)-2s_L-v\delta}{2(4-\delta)};$$
$$p_H - p_L - (s_H - s_L) = \frac{2s_L(3-\delta)-3s_H(2-\delta)-v\delta}{2(4-\delta)}.$$

Then, $p_H - p_L - (s_H - s_L) > 0$ if $s < \frac{2s_L(3-\delta)-v\delta}{3(2-\delta)} = s_L - \frac{\delta(v-s_L)}{3(2-\delta)} < s_L$,which is a contradiction. Therefore, $p_H - p_L < s_H - s_L$.

Endogenous case:

1. When $\tau < \tau^{IC} = \frac{(2(s_H-s_L)-(v-s_H)\delta)\delta}{4(2-\delta)}$;

$$\begin{aligned} p_H - p_L &= \frac{\tau}{\delta}, \\ p_H - p_L - (s_H - s_L) &> 0 \text{ if } \tau > (s_H - s_L)\delta. \end{aligned}$$

Therefore, for the condition to hold, it must be the case that

$$\tau^{IC} > (s_H - s_L)\delta.$$

However, we can easily see that $\tau^{IC} \leq (s_H - s_L)\delta$ because

$$\begin{aligned} \tau^{IC} - (s_H - s_L)\delta &= -\frac{\delta(s_H(6-5\delta)+v\delta-s_L(6-4\delta))}{4(2-\delta)} \\ &= -\frac{\delta((s_H-s_L)(6-5\delta)+\delta(v-s_L))}{4(2-\delta)} \leq 0. \end{aligned}$$

Therefore, $p_H - p_L < s_H - s_L$.

2. When $\tau > \tau^{IC} = \frac{(2(s_H-s_L)-(v-s_H)\delta)\delta}{4(2-\delta)}$;

$$p_H - p_L = \frac{s_H(2+\delta)-2s_L-v\delta+2\tau}{8-2\delta}.$$

We first note that $p_H - p_L - (s_H - s_L) > 0$ if

$$\begin{aligned}\tau &> \tau^* = \frac{s_H(6-3\delta) + v\delta - s_L(6-2\delta)}{2} \\ &= \frac{s_H(6-3\delta) + v\delta - s_L\delta - s_L(6-3\delta)}{2} \\ &= \frac{(s_H - s_L)(6-3\delta) + \delta(v - s_L)}{2}.\end{aligned}$$

And we can show that $\tau^* > \tau^{IC}$ since

$$\tau^* - \tau_{IC} = \frac{(4-\delta)((s_H - s_L)(6-5\delta) + \delta(v - s_L))}{4(2-\delta)} > 0.$$

Therefore,

$$p_H - p_L > s_H - s_L \text{ if } \tau > \tau^* = \frac{(s_H - s_L)(6-3\delta) + \delta(v - s_L)}{2}.$$