

## **Technical Appendix**

A Customer Management Dilemma:  
When is it profitable to reward own customers?

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## **Technical Appendix: Table of Contents**

- 1. Modeling the interplay between quantity and loyalty**
- 2. Comparison of average prices between own and competitor's customers**
- 3. Numerical examples**

## Technical Appendix

### 1. Modeling the interplay between quantity and loyalty

We extend our model to incorporate the interplay between quantity and loyalty. In our basic model, we assumed that both high and low type customers have the same level of preference stochasticity. Hence, there is no explicit relationship between purchase quantity and customer loyalty. However, it is possible that preference stochasticity differs across the high and low segments due to differences in loyalty. The fact that some consumers buy larger quantities can be a sign that they are more loyal to the firm. To assess the robustness of our model to this potential relationship between quantity and loyalty, we extend our basic model to allow for different levels of preference stochasticity for each segment. We consider a special case of our general model, where  $\alpha = 0.5$ ,  $q = 2$  case as an example for this robustness analysis.

We capture differences in preference stochasticity between the two segments using a multiplier,  $\kappa \in [0,1]$ , where  $H$ -type customers' preference stochasticity is now  $\kappa \cdot \beta$  and the  $L$ -type customers preference stochasticity is  $\beta$ . The  $\kappa$  implies that high type customers will on average be more loyal to the current supplier (i.e., they are less vulnerable to the external shock  $\beta$ ) in the second period.

Thus, second-period locations may change due to external situational shock, drawn from a uniform distribution ( $\omega \sim U[0,1]$ ) with probability  $\beta$  for low type customers and  $\kappa \cdot \beta$  for high type customers, where  $\beta \in [0,1]$  captures the extent of the preference stochasticity across time. Therefore, customers' locations correlate over the two periods but high type customers are more likely to stay in the same location (more likely to be loyal), and the expected location of the second period for a customer is  $E^H(\theta_2) = \kappa\beta\omega + (1 - \kappa\beta)\theta_1$  and  $E^L(\theta_2) = \beta\omega + (1 - \beta)\theta_1$  for high type and low type customer, respectively.

We can now re-write the conditional probability that  $H$ -type customers will locate in a certain range of the Hotelling line, given their first-period purchase choice of  $A$  or  $B$ , as follows:

$$\Pr[\theta_2 \leq x | \theta_1 \leq \tilde{\theta}_1] = \begin{cases} (1 - \kappa \cdot \beta) + \kappa \cdot \beta x & \text{if } \tilde{\theta}_1 \leq x, \\ (1 - \kappa \cdot \beta) \frac{x}{\tilde{\theta}_1} + \kappa \cdot \beta x & \text{if } \tilde{\theta}_1 > x, \end{cases} \quad (\text{T1})$$

$$\Pr[\theta_2 > x | \theta_1 > \tilde{\theta}_1] = \begin{cases} (1-l \cdot \beta) + l \cdot \beta(1-x) & \text{if } \tilde{\theta}_1 \geq x, \\ (1-l \cdot \beta) \frac{1-x}{1-\tilde{\theta}_1} + l \cdot \beta(1-x) & \text{if } \tilde{\theta}_1 < x. \end{cases} \quad (\text{T2})$$

Similar to the analysis in the text, the second-period profits of retailers  $A$  and  $B$  in equation (3) are now re-written as

$$\begin{aligned} \Pi_2^A &= (p_2^{AH})q\tilde{\theta}_1^H \Pr^{AH} + (p_2^{AL})\tilde{\theta}_1^L \Pr^{AL} + (p_2^{AO}) \left\{ q(1-\tilde{\theta}_1^H)(1-\Pr^{BH}) + (1-\tilde{\theta}_1^L)(1-\Pr^{BL}) \right\} \\ \Pi_2^B &= (p_2^{BH})q(1-\tilde{\theta}_1^H) \Pr^{BH} + (p_2^{BL})(1-\tilde{\theta}_1^L) \Pr^{BL} + (p_2^{BO}) \left\{ q\tilde{\theta}_1^H (1-\Pr^{AH}) + \tilde{\theta}_1^L (1-\Pr^{AL}) \right\} \end{aligned} \quad (\text{T3})$$

Using equations (T1) and (T2) as well as equations (1) and (2) in text, we obtain the second-period prices by solving the retailers' first-order conditions.

We can replicate our main result about rewarding own high type customers when there is a threshold level of preference stochasticity among the  $H$ -type (i.e.,  $\kappa \geq \frac{-9+25\alpha+\sqrt{81-66\alpha+241\alpha^2}}{32\alpha}$ ).

We state this finding in the following proposition:

**Proposition T1:**

Suppose that  $\kappa \geq \frac{-9+25\alpha+\sqrt{81-66\alpha+241\alpha^2}}{32\alpha}$  (i.e., in this special case of  $\alpha = \frac{1}{2}$ ,  $\kappa \geq \frac{7+\sqrt{433}}{32} \approx 0.869$ ).

**(a) Reward Competitor's Customers:** When customer mobility is low ( $\beta < \underline{\chi}(2, \frac{1}{2}, \kappa) = \frac{18}{5+16\kappa}$ ), there exists a symmetric pure strategy equilibrium in second-period prices, such that retailers charge  $p_2^{AH} = p_2^{BH} = \frac{42-8\kappa\beta-\beta}{12(10-4\kappa\beta-\beta)}$ ,  $p_2^{AL} = p_2^{BL} = \frac{18-5\kappa\beta-\beta}{3(10-4\kappa\beta-\beta)}$ , and  $p_2^{AO} = p_2^{BO} = \frac{6+2\kappa\beta+\beta}{3(10-4\kappa\beta-\beta)}$ . The equilibrium second-period profits are  $\Pi_2^A = \Pi_2^B = \frac{756-\beta(66-8\beta+\kappa(144-(41+32\kappa)\beta))}{288(10-4\kappa\beta-\beta)}$ .

Moreover, prices follow an ordinal relationship,  $p_2^{iO} \leq p_2^{iH} \leq p_2^{iL}$ , where  $i \in \{A, B\}$ ; that is, a competitor's customers receive the lowest price.

**(b) Reward Own Customers:** When customer mobility is sufficiently high ( $\beta \geq \bar{\chi}(2, \frac{1}{2}, \kappa) = \frac{3(\kappa-1)+\sqrt{3(67\kappa^2-10\kappa+3)}}{16\kappa^2-1}$ ) and consumer heterogeneity in purchase quantity exists

( $q=2$ ), there exists a symmetric pure strategy equilibrium in second period such that retailers charge  $p_2^{AH} = p_2^{BH} = \frac{1}{12} \left( \frac{6}{\kappa\beta} + \frac{2(6+\beta+2\kappa\beta)}{2-(1-4\kappa)\beta} - 3 \right)$ ,  $p_2^{AL} = p_2^{BL} = \frac{6-\beta+7\kappa\beta}{6-3\beta+12\kappa\beta}$ , and

$$p_2^{AO} = p_2^{BO} = \frac{6+\beta+2\kappa\beta}{6-3\beta+12\kappa\beta}. \quad \text{The second-period profits are } \Pi_2^A = \Pi_2^B = \frac{752+\beta(-36+\kappa(504+\beta(60+66\kappa+\beta(8+\kappa(32\kappa-49))))))}{288\kappa\beta(2+(4\kappa-1)\beta)}.$$

Moreover, prices follow an ordinal relationship,  $p_2^{iH} \leq p_2^{iO} \leq p_2^{iL}$ , where  $i \in \{A, B\}$ ; that is, the retailer's own high-type customers receive the lowest price.

**Proof.**

First, we start with the observation that when  $\kappa \geq \frac{7+\sqrt{433}}{32} \approx 0.869$ ,  $\underline{\chi}(2, \frac{1}{2}, \kappa) = \frac{18}{5+16\kappa} < \overline{\chi}(2, \frac{1}{2}, \kappa) = \frac{3(\kappa-1)+\sqrt{3(67\kappa^2-10\kappa+3)}}{16\kappa^2-1} \leq 1$ . Hence, there always exists  $\beta < \underline{\chi}(2, \frac{1}{2}, \kappa) = \frac{18}{5+16\kappa} < \overline{\chi}(2, \frac{1}{2}, \kappa) = \frac{3(\kappa-1)+\sqrt{3(67\kappa^2-10\kappa+3)}}{16\kappa^2-1} \leq 1$  when  $\alpha = 0.5$ ,  $q = 2$ .

(a) First, consider the case when  $\beta < \underline{\chi}(2, \frac{1}{2}, \kappa) = \frac{18}{5+16\kappa}$ , which ensures that  $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$  and  $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$ , in equilibrium. Again, using equations (1) and (2), we know that  $\Pr^{AH} = \left(\frac{1-\kappa\beta}{\tilde{\theta}_1^H} + \kappa\beta\right)\left(\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right)$ ,  $\Pr^{AL} = \left(\frac{1-\beta}{\tilde{\theta}_1^L} + \beta\right)\left(\frac{1+p_2^{Bo}-p_2^{AL}}{2}\right)$ ,  $\Pr^{BH} = \left(\frac{1-\kappa\beta}{1-\tilde{\theta}_1^H} + \kappa\beta\right)\left(\frac{1-q(p_2^{BH}-p_2^{Ao})}{2}\right)$ , and  $\Pr^{BL} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right)\left(\frac{1-p_2^{BL}+p_2^{Ao}}{2}\right)$ . Similar to the proof of Proposition 1, we solve the first-order conditions. When  $\tilde{\theta}_1^j = \frac{1}{2}$ , second-period prices are  $p_2^{AH} = p_2^{BH} = \frac{42-8\kappa\beta-\beta}{12(10-4\kappa\beta-\beta)}$ ,  $p_2^{AL} = p_2^{BL} = \frac{18-5\kappa\beta-\beta}{3(10-4\kappa\beta-\beta)}$ , and  $p_2^{AO} = p_2^{BO} = \frac{6+2\kappa\beta+\beta}{3(10-4\kappa\beta-\beta)}$ , which confirms that  $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$  and  $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$  when  $\beta < \underline{\chi}(2, \frac{1}{2}, \kappa) = \frac{18}{5+16\kappa}$ . It is obvious to show that  $p_2^{iO} \leq p_2^{iH} \leq p_2^{iL}$  from a direct comparison. ■

(b) Next, we look at the case  $\beta \geq \overline{\chi}(2, \frac{1}{2}, \kappa) = \frac{3(\kappa-1)+\sqrt{3(67\kappa^2-10\kappa+3)}}{16\kappa^2-1}$ , which ensures that in equilibrium, the market is  $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$  and  $\tilde{\theta}_1^H \leq \tilde{\theta}_2^{AH}$ . Again, from equation (1) and (2) we know that  $\Pr^{AH} = (1-\kappa\beta) + \kappa\beta\left(\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right)$ ,  $\Pr^{AL} = \left(\frac{1-\beta}{\tilde{\theta}_1^L} + \beta\right)\left(\frac{1+p_2^{Bo}-p_2^{AL}}{2}\right)$ ,  $\Pr^{BH} = (1-\kappa\beta) + \kappa\beta\left(\frac{1-q(p_2^{BH}-p_2^{Ao})}{2}\right)$ , and  $\Pr^{BL} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right)\left(\frac{1-p_2^{BL}+p_2^{Ao}}{2}\right)$ . Plugging these into equation (3), we obtain the second-period prices by solving the retailers' first-order conditions. Specifically, when  $\tilde{\theta}_1^j = \frac{1}{2}$ , the second-period prices are

$$p_2^{AH} = p_2^{BH} = \frac{1}{12} \left( \frac{6}{\kappa\beta} + \frac{2(6+\beta+2\kappa\beta)}{2-(1-4\kappa)\beta} - 3 \right), \quad p_2^{AL} = p_2^{BL} = \frac{6-\beta+7\kappa\beta}{6-3\beta+12\kappa\beta}, \quad \text{and} \quad p_2^{AO} = p_2^{BO} = \frac{6+\beta+2\kappa\beta}{6-3\beta+12\kappa\beta}. \quad \text{We}$$

confirm that  $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$  and  $\tilde{\theta}_1^H \leq \tilde{\theta}_2^{AH}$  when  $\beta \geq \frac{3(\kappa-1) + \sqrt{3(67\kappa^2 - 10\kappa + 3)}}{16\kappa^2 - 1} = \bar{\chi}(2, \frac{1}{2}, \kappa)$ . It is obvious

to show that  $p_2^{iH} \leq p_2^{iO} \leq p_2^{iL}$  from a direct comparison. ■

### **Q.E.D.**

In Proposition T1, we replicate our main result. Even if the high volume type is more loyal (i.e., more stable preference and has  $\kappa\beta$  while the low type has preference stochasticity of  $\beta$ ), our result about rewarding high volume customers holds beyond a certain threshold level of  $\kappa = \frac{7+\sqrt{433}}{32}$ . However, if  $\kappa$  is below the threshold (i.e., high preference stability), then clearly the firm has no incentive to reward its own high type customers because they are very likely to stay with the firm. For example, when  $\kappa < \frac{7+\sqrt{433}}{32}$ , firms only reward competitor's customers.

## 2. Comparison of average prices between own and competitor's customers

We show the condition where the average price,  $p_2^{i-avg} = \frac{p_2^{iH} + p_2^{iL}}{2}$ , offered to own customers is lower than the price for the competitor's customers.

### Proposition T2:

Suppose that both heterogeneity in quantities ( $q \geq 2$ ) and high levels of preference stochasticity

exist ( $\beta \geq \bar{\chi}(\alpha, q) = \frac{2\alpha q^2 - (q+6)(1-\alpha) + \sqrt{(2\alpha q^2 - (q+6)(1-\alpha))^2 + 12(1-\alpha)(4\alpha q^2 + (q-3)(1-\alpha))}}{4\alpha q^2 + (q-3)(1-\alpha)}$ ).

If the portion of  $H$ -type customer is sufficiently small, then the average price offered to own customers,  $p_2^{i-avg} = \frac{p_2^{iH} + p_2^{iL}}{2}$  can be lower than the price for the competitor's customers.

### Proof.

From Proposition 1, we know that  $\beta \geq \bar{\chi}(\alpha, q) = \frac{2\alpha q^2 - (q+6)(1-\alpha) + \sqrt{(2\alpha q^2 - (q+6)(1-\alpha))^2 + 12(1-\alpha)(4\alpha q^2 + (q-3)(1-\alpha))}}{4\alpha q^2 + (q-3)(1-\alpha)}$ ,

and there exists customer heterogeneity in purchase quantity ( $q \geq 2$ ), then it is optimal for firms to reward own best customers.

The equilibrium prices are  $p_2^{AH} = \frac{2-\beta}{2q\beta} + \frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)}$ ,  $p_2^{AL} = \frac{1}{2} + \frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)}$ , and

$p_2^{AO} = \frac{(2+\beta)(1+(q-1)\alpha)}{3((2-\beta)(1-\alpha)+q^2\alpha\beta)}$ . The average prices are, therefore,

$$p_2^{i-avg} = \frac{p_2^{iH} + p_2^{iL}}{2} = \frac{1}{12} \left( 3 + \frac{6-3\beta}{q\beta} + \frac{2(1+(-1+q)\alpha)(2+\beta)}{2-\beta+\alpha(-2+\beta+q^2\beta)} \right).$$

Now, we can compare the average price of own customers ( $p_2^{i-avg}$ ) and average price of competitor's customers ( $p_2^{AO}$ ). After a few algebraic steps, we can get

$$p_2^{i-avg} - p_2^{AO} = \frac{-12(-1+\alpha) + 2(-6+q+(6+(-1+q)q)\alpha)\beta + (3-5q+(-1+q)(3+q(-2+3q))\alpha)\beta^2}{12q\beta(2-\beta+\alpha(-2+\beta+q^2\beta))} > 0$$

$$\Leftrightarrow \alpha \leq \frac{-12+12\beta-2q\beta-3\beta^2+5q\beta^2}{-12+12\beta-2q\beta+2q^2\beta-3\beta^2+5q\beta^2-5q^2\beta^2+3q^3\beta^2}$$

In particular, when  $\beta=1, q=2$ , the above equation simplifies to

$$p_2^{i-avg} - p_2^{AO} = \frac{-7-12(-1+\alpha)+11\alpha+2(-4+8\alpha)}{24(1+3\alpha)} > 0$$

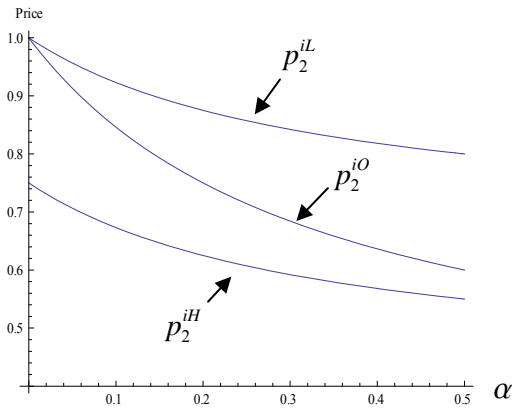
$$\Leftrightarrow \alpha \leq \frac{1}{5}.$$

**Q.E.D.**

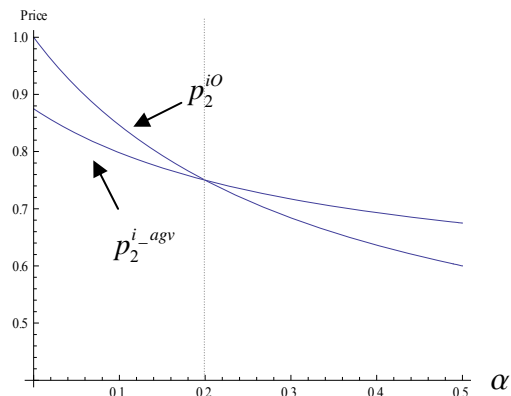
Proposition T2 states that when both heterogeneity in quantities and high levels of preference stochasticity exist (so that the price for  $H$ -type is sufficiently low), and the portion of  $H$ -type customer is sufficiently small, then the average price offered to own customers,  $p_2^{i-avg} = \frac{p_2^{iH} + p_2^{iL}}{2}$ , can be lower than the price for the competitor's customers. This is so because the poaching price (i.e., the average price of competitor's customers) becomes much larger (note that the poaching price increases as  $\alpha$  decreases) while the price for  $H$ -type customers is lower. We illustrate this in the following Figure T1, where  $\beta=1, q=2$ . In this figure, the  $x$ -axis is  $\alpha$ . We fix the preference stochasticity as 1 (the most favorable case) to illustrate the intuition.

Figure T1: Average prices when  $\beta = 1$  and  $q = 2$

(1) Own  $H$ -type,  $L$ -type, and poaching prices



(2) Average prices between own customers and competitor's customers



As Figure T1 (1) demonstrates, as  $\alpha$  becomes close to zero, the poaching price increases significantly while the price for own  $H$ -type customers increase mildly. Hence, the average price for own customers becomes lower than the average price for competitor's customers as shown in Figure T1 (2).

### 3. Numerical Examples

We elucidate the implications in propositions 1 and 3 of the paper with numerical examples in Table 2 for alternative levels of preference stochasticity to illustrate both cases of rewarding own best customers and rewarding competitor's customers when  $\alpha = 0.5, q = 2$ .

Table 2: Numerical Illustration ( $\alpha = \frac{1}{2}, q = 2$ )

	$\beta = 0$	$\beta = 0.5$	$\beta = 0.9$	$\beta = 1$
1 <sup>st</sup> period price	$p_1^A = \frac{4}{5}$	$p_1^A = \frac{125}{132}$	$p_1^A = 0.575$	$p_1^A = \frac{13597}{27000} \approx 0.504$
2 <sup>nd</sup> period prices	$p_2^{AH} = \frac{7}{20}$ $p_2^{AL} = \frac{3}{5}$ $p_2^{AO} = \frac{1}{5}$	$p_2^{AH} = \frac{5}{12}$ $p_2^{AL} = \frac{2}{3}$ $p_2^{AO} = \frac{1}{3}$	$p_2^{AH} = \frac{1039}{1692} \approx 0.614$ $p_2^{AL} = \frac{38}{47} \approx 0.809$ $p_2^{AO} = \frac{29}{47} \approx 0.617$	$p_2^{AH} = \frac{11}{20}$ $p_2^{AL} = \frac{4}{5}$ $p_2^{AO} = \frac{3}{5}$
Total Profit	$\Pi^{BBP} = \Pi_1 + \Pi_2$ $= \frac{3}{5} + \frac{21}{80}$ $= \frac{69}{80} \approx 0.863$	$\Pi^{BBP} = \Pi_1 + \Pi_2$ $= \frac{125}{176} + \frac{61}{192}$ $\approx 1.028$	$\Pi^{BBP} = \Pi_1 + \Pi_2$ $= 0.431 + 0.48$ $\approx 0.915$	$\Pi^{BBP} = \Pi_1 + \Pi_2$ $= \frac{13597}{36000} + \frac{73}{160}$ $\approx 0.835$
Profit without BBP	$\Pi^{\text{No BBP}} = 0.9$	$\Pi^{\text{No BBP}} = 0.9$	$\Pi^{\text{No BBP}} = 0.9$	$\Pi^{\text{No BBP}} = 0.9$
Comparison	$p_2^{AO} < p_2^{AH} < p_2^{AL}$ $\Pi^{\text{No BBP}} > \Pi^{BBP}$	$p_2^{AO} < p_2^{AH} < p_2^{AL}$ $\Pi^{\text{No BBP}} < \Pi^{BBP}$	$p_2^{AH} < p_2^{AO} < p_2^{AL}$ $\Pi^{\text{No BBP}} < \Pi^{BBP}$	$p_2^{AH} < p_2^{AO} < p_2^{AL}$ $\Pi^{\text{No BBP}} > \Pi^{BBP}$
	Reward <b>Competitor's</b> Customers BBP <b>reduces profits</b>	Reward <b>Competitor's</b> Customers BBP <b>increases profits</b>	Reward <b>Own Best</b> Customers BBP <b>increases profits</b>	Reward <b>Own Best</b> Customers BBP <b>reduces profits</b>